LATERAL VIBRATION PREDICTION ISSUES

Dr. Yuriy Batrak

1. Do we need to predict lateral vibration?

One third of ship problems by cost for 1998-2006 is related to machinery, Fig.1.1.

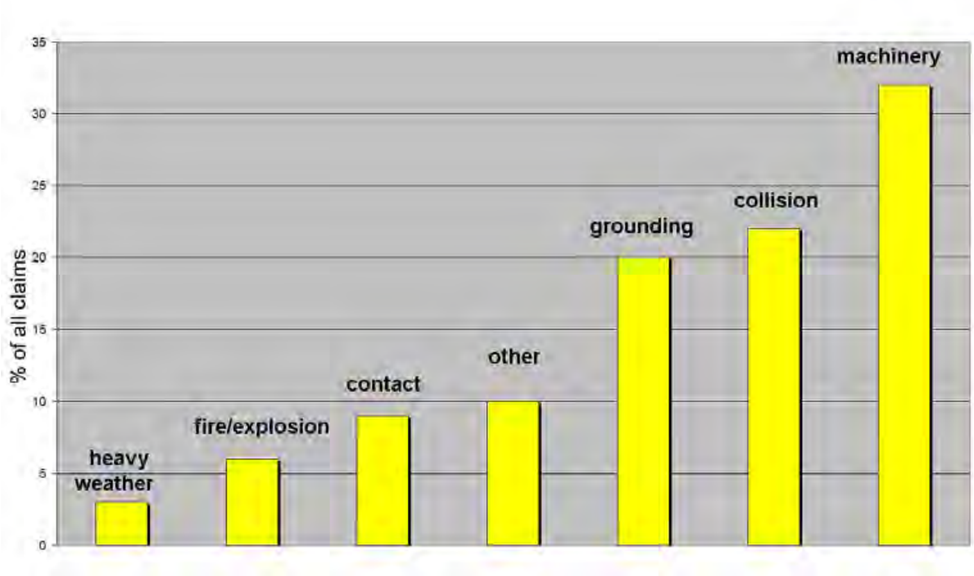

Fig. 1.1 Hull and machinery claims by cost [1]

Propulsion claims stands on the second place after main engine claims and are nearly one fourth of the machinery claims, (Fig.1.2).

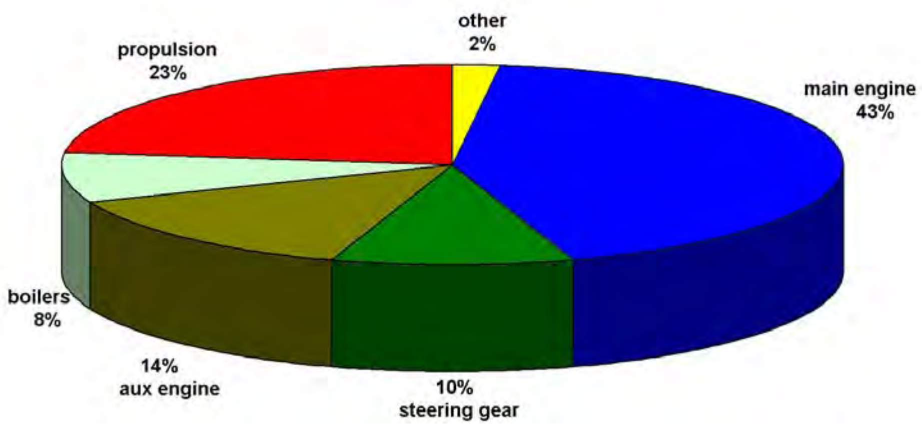

Fig. 1.2 Machinery claims by costs [1]

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Without any doubts whirling vibration, owing to fluctuating propeller forces, system unbalance, fluctuations induced by the main engine, makes a contribution to the annoying statistics.
Excessive whirling vibration adversely influences particular propulsion system elements and system integrity in whole.
The problems caused by whirling vibration are as the following:

- complete shafting system destruction;
- shaft and shaft elements fatigue life reduction;
- fatigue cracks at shaft brackets and foundations;
- stern tube bearing fatigue damage;
- increased wear and damage of the sealing;
- excessive noise, hull and superstructure vibrations.

Completely destroyed high speed (1800 rpm) propulsion shafting with cardan shafts is shown on Fig.1.3. Relation L/d (span/diameter) for one of the shafts was greater than 20 instead of the recommended lesser than 20.

Fig. 1.3 Destroyed high speed shaft line with cardan shafts

As per statistics most subjected to propulsion shafting damage are tugs and supply vessels, Fig.1.4.

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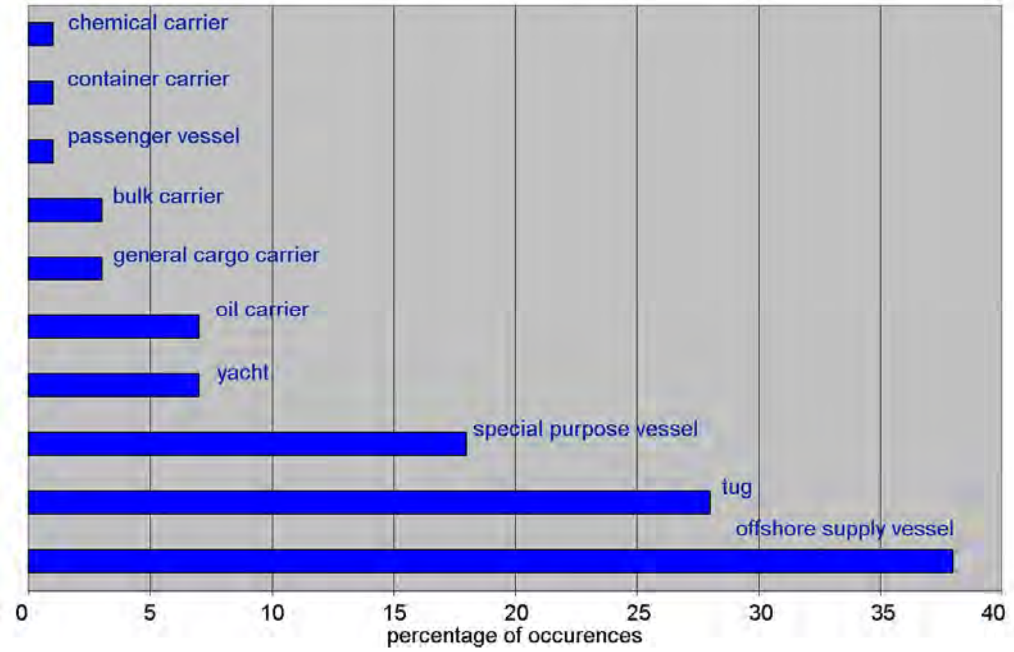

Fig. 1.4 Propulsion shafting damage frequency distribution by ship type [1]

Both ship types have the same operation profile, with frequent switching from ahead to astern operation that promotes fatigue and other vibration related damages.
Aft stern tube bearing is the most vulnerable element of the propulsion system. Number of the stern bearings damages fixed yearly is shown on Fig. 1.5.

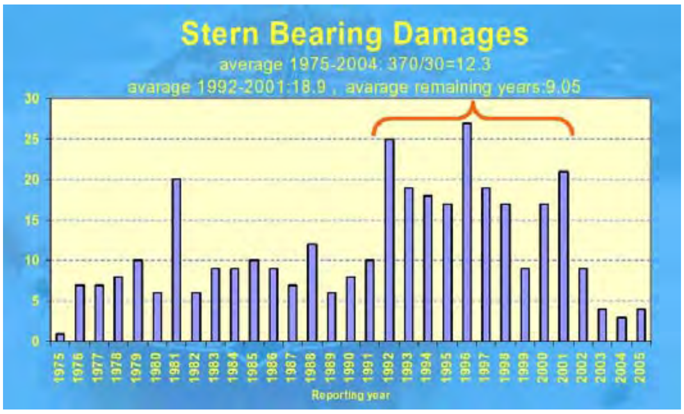

Fig. 1.5 DNV statistics of aft stern tube bearing damages

There are two typical damages of stern tube bearings: wiping and bearing material fatigue caused by fluctuating pressure and cyclic loading [2]. The cause of the last one is a propeller shaft whirling. Destroyed white metal liner of the stern tube bearing is shown Fig. 1.6.

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Fig. 1.6 Fatigue damage of the stern tube bearing [3]

Increased whirling vibration speeds up wearing process of the sealing and promotes sea water pollution with oil, Fig. 1.7.

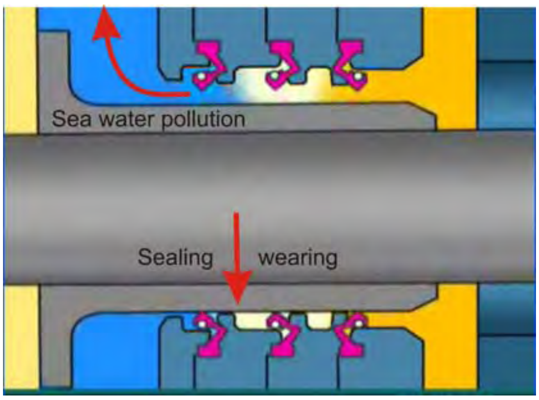

Fig. 1.7 Sealing wearing and sea water pollution

Partial propeller immersion increases shaft vibrations and affects stern tube bearings much more. As can be seen from Fig.I.8 [4] vibration amplitudes of the tail shaft with a partially immersed propeller grow significantly when rotation speed increases.

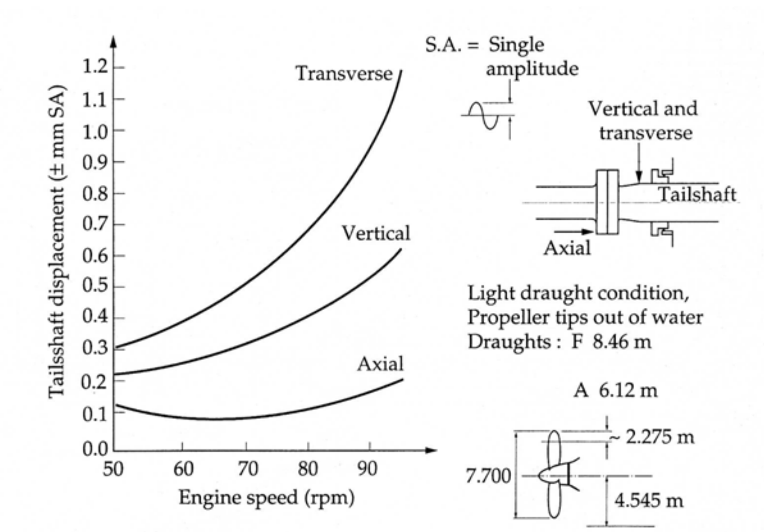

Fig. 1.8 Vibration amplitudes of the tail shaft with partially immersed propeller

2. Lateral, bending, transverse or whirling vibration?

2.1 Definitions variety

Natural language has a remarkable feature. It is very conservative. It holds the marks of distant events and reflects the modern world reality.
When we open the Rules of different Classification Societies IACS members, we can find different names for designation of the same subject:

Classification Society	Term used
American Bureau of Shipping (ABS)	– Lateral (Whirling) vibration
Bureau Veritas (BV)	– Bending vibration
China Classification Society (CCS)	– Whirling vibration
Det Norske Veritas (DNV)	– Whirling vibration, Lateral vibration
Germanischer Lloyd (GL)	– Bending vibration
Indian Register of Shipping (IRS)	– Lateral vibration
Korean (KR)	– Bending vibration
Lloyd's Register of Shipping (LR)	– Lateral vibration
Nippon Kaiji Kiokai (NK)	– Lateral vibration
RINA	– Bending vibration
Russian Maritime Register (RS)	– Bending vibration

There is no doubt that three terms 'lateral vibration', 'bending vibration' 'whirling vibration' used to designate the same type of propulsion shaft system motion. Sometimes the term 'transverse vibration' is used too. The variety of used terms is a reflection of a certain ambiguity of this type vibration definition.

In contrast to the torsional vibration for which all calculation circumstance and acceptance criteria are well defined by Classification Societies Rules, lateral (bending, whirling) vibration requirements are formulated shortly and in general terms or not formulated at all.

It is well known that lateral (bending, whirling, transverse) vibration of propulsion systems phenomenon is not as dangerous as the torsional vibration. In general dynamic bending stresses in propulsion system are negligible in comparison with the torsional vibration stress. In addition the rigorous calculation of propulsion system lateral (bending, whirling, transverse) vibration is much more difficult problem than torsional vibration calculation.

As a consequence propulsion system lateral (bending, whirling, transverse) vibration issues remains are not thoroughly investigated and this situation is reflected in terms variety.

2.2 Lateral vibration

The Webster dictionary defines term 'lateral' as 'relating to the side' or 'coming from the side'. We say that beam lateral vibration occurs when the direction of beam motion is normal to the centerline of a beam.

However to imagine the propulsion system lateral vibration we never think about solid body lateral motions. The first that we think about is a beam bending deformations when beam points move from side to side up and down due to the shaft flexibility, Fig.2.1.

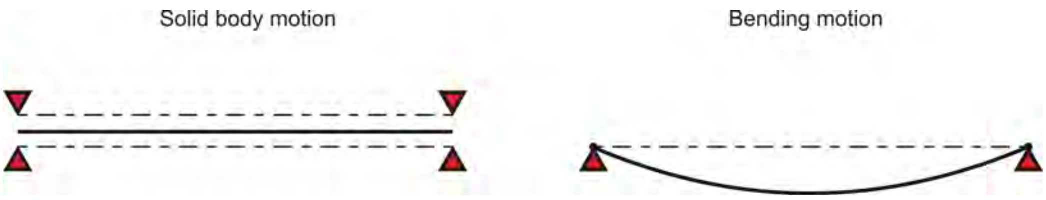

Fig.2.1 Single span uniform beam lateral motions

So if we do not take into account beam solid body motions terms 'lateral vibration' 'bending vibration' and 'transverse vibration' are the synonyms. Terms 'lateral' and 'transverse' accentuate the beam motions, term 'bending' emphasizes the beam flexibility phenomenon.

As far as lateral solid body motions of the shafts lying on the plain bearings with a certain clearance are not excluded we do prefer use most general term 'lateral vibration'.

The lateral vibration equation used in FEM calculation is as follows:

$$\mathbf{M}\ddot{\mathbf{X}}(t)+\mathbf{C}\dot{\mathbf{X}}(t)+\mathbf{K}\mathbf{X}(t)=\mathbf{F}(t),$$

where: $\mathbf{X}(t)$ – solution of the equation (deflection and slopes at the system nodes);

$\mathbf{M}$ – mass matrix;
$\mathbf{C}$ – damping matrix;
$\mathbf{K}$ – stiffness matrix;
$\mathbf{F}$ – excitation forces vector.

In a weakly damped system damping forces does not influence free vibration frequencies and mode shapes very much, therefore free vibration problem can be solved using the equation for non damped vibration:

$$\mathbf{M}\ddot{\mathbf{X}}(t)+\mathbf{K}\mathbf{X}(t)=\mathbf{0}.$$

Non damped natural frequencies and bending mode shapes of a single span uniform beam are shown on, Fig.2.2.

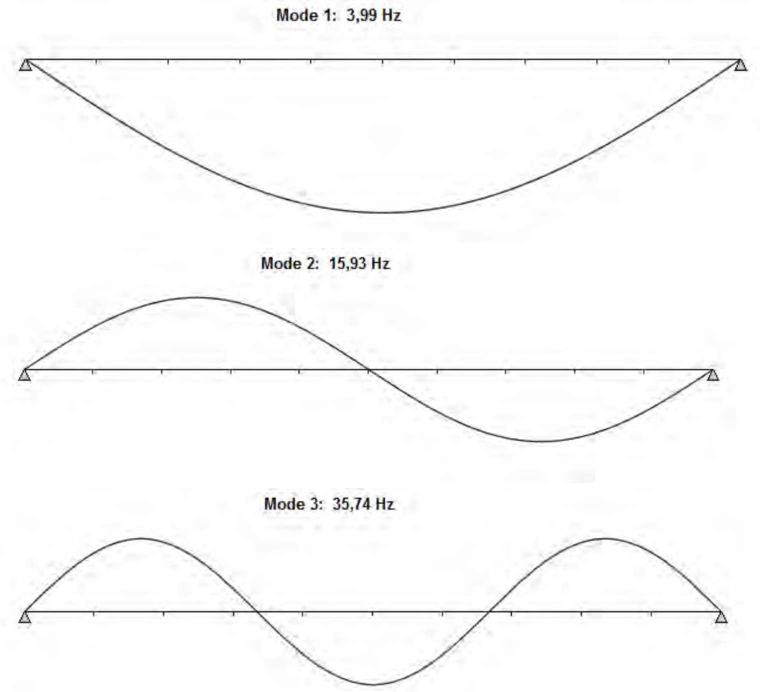

Fig.2.2 Natural frequencies and bending mode shapes of the single span beam

2.3 Vibration glossary

Free vibration – occurs when $\mathbf{F}(t)=\mathbf{0}$ i.e. a mechanical system vibrates freely after an initial motion was applied.

Forced vibration – occurs where an alternating force $\mathbf{F}(t)\neq\mathbf{0}$ is applied to a mechanical system. In forced vibration the alternating force $F(t)$ does not disappear when the excited motion is prevented.

Self-excited vibration – occurs where the alternating force $\mathbf{F}(t)$ that sustains the vibration motion is created or controlled by the vibration motion itself. When the motion stops the alternating force $\mathbf{F}(t)$ disappears.

Steady vibration – vibration of a mechanical system caused by a periodic excitation when free vibration oscillations have decayed.

Harmonic excitation – occurs when the periodic excitation force alternates according to the harmonic law: $f(t)=A\sin(\omega t+\psi)$.

Transient vibration – occurs when a non periodic alternating force $\mathbf{F}(t)$ is applied.

Parametric vibration – occurs when mass $\mathbf{M}$ and/or damping $\mathbf{C}$ and/or stiffness $\mathbf{K}$ of a mechanical system are variable ($\mathbf{M}(t), \mathbf{C}(t), \mathbf{K}(t)$), but not depend on vibration motion.

Non liner vibration – occurs when mass $\mathbf{M}$ and/or damping $\mathbf{C}$ and/or stiffness $\mathbf{K}$ of a mechanical system depend on vibration motion $\mathbf{X}(t)$.

Non damped vibration – occurs when damping in vibrating system is equal to zero $\mathbf{C}=\mathbf{0}$.

When excitation force frequency coincides with one of the natural frequencies the violent resonance vibration occurs. At the resonance frequency beam restoring forces increase at the same rate as the inertia forces, so they cancel each other out. Excitation force remains not compensated. Vibration amplitude in non damped system increases ad

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infinitum, but in the damped system it remains finite because the excitation force is cancelled by the damping forces.

Formulated above lateral vibration definitions and solutions is applicable not only to the beam vibrating in one plane only, it can be applied to the beams vibrating in both vertical and horizontal planes simultaneously, Fig.2.3.

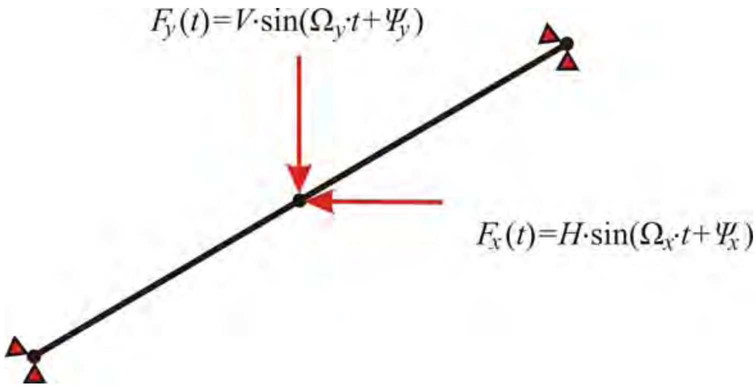

Fig.2.3 Single span beam excited in two planes simultaneously

Steady non damped forced vibration in two planes of a single span uniform beam is a polyharmonical motion, which is a sum of two harmonic motions. Coordinates of the beam centerline point in such motion are defined as:

$$X(t) = A_x sin(\Omega_x t + \psi_x),$$

$$Y(t) = A_y sin(\Omega_y t + \psi_y),$$

where:

$A_x,\ A_y$ – amplitudes of vibration;

$\Omega_x,\ \Omega_y$ – frequencies of vibration;

$\psi_x,\ \psi_y$ – phases of vibration.

Centerline points of the beam, vibrating simultaneously in two orthogonal planes, move along the curves called as ‘orbits’. Shaft centerline point orbits have a shape of Lissajou’s figures depending on frequencies relation and phase difference $\delta = \psi_x - \psi_y$ of the motions (Fig.2.4).

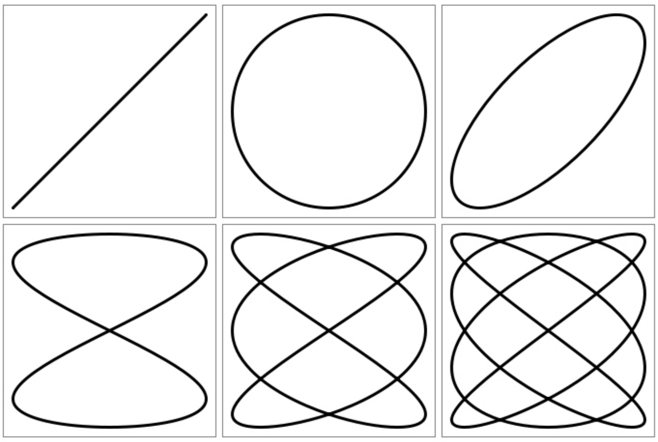

Fig.2.4 Beam centerline orbits of polyharmonical vibration

When the relation of frequencies Ω_x / Ω_y is a rational number all the shapes are line segments or closed curves.
The orbit shape will looks like:

– a line segment if $\Omega_x = \Omega_y$ and $\delta = 0$ or $\delta = \pi$,

– an ellipse if $\Omega_x = \Omega_y$, $\delta = \pi / 2$,

– a circle if $\Omega_x = \Omega_y$, $A_x = A_y$, and $\delta = \pi / 2$.

When beam centerline orbit shape is an ellipse or a circle, it moves around geometry axis with a constant angular speed $\Omega = \Omega_x = \Omega_y$. This motion is plane-parallel, i.e. section does not rotate around the cross section center, Fig.2.5.

It is important to note that depending on phase difference δ speed value Ω can be positive as well as negative i.e. the orbit can be oriented clockwise or counter clockwise.

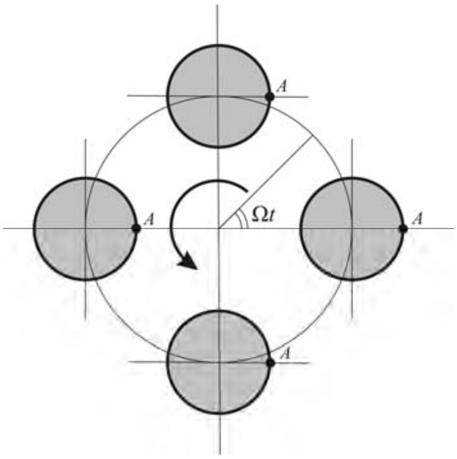

Fig.2.5 Beam section motion in polyharmonical vibration

2.4 Whirling vibration

Every point of a flexible rotating shaft moves with angular speed ω around the tangent to deflection curve of the shaft, Fig.2.6.

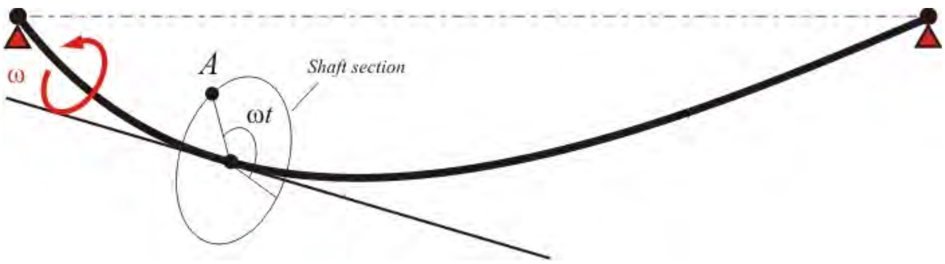

Fig.2.6 Rotation of the flexible shaft

At the same time there are no obstacles for rotating shaft to vibrate as above-mentioned beam in two orthogonally related planes if correspondent alternating forces are applied. As a result the shaft rotation axis will move in the space along the beam vibration orbit.

Change in the position of the rotation axis of a rotating body in classical mechanics is known as a ***precession*** motion. Conventional name for the precession motion of the rotating shafts is '***whirling'***. When precession orbit shape is circular or elliptical we have the case of a circular or elliptical whirling. Very often term 'whirling' associate with a circular or elliptical orbit shape exclusively, but it is not quite correct from general point of view.

Whirling speed is equal to $\Omega = \sqrt{\Omega_x^2 + \Omega_y^2}$. It should be specially emphasized that in general case whirling speed Ω and its direction by no means depends on value and direction of shaft angular speed ω.

When the directions of shaft rotation speed ω and circular or elliptical whirling speed Ω coincide, whirling is denominated as a ***forward whirling***; if the directions are opposite, a circular or elliptical whirling is denominated as a ***backward whirling***. When $|\Omega| = |\omega|$ it is the case of a ***synchronous whirling***. It is the case of an ***asynchronous whirling*** if $|\Omega| \neq |\omega|$.

Thus whirling is a polyharmonical motion of the rotating shaft caused by two plane excitation forces. Every time when 'whirling vibration' term is used with regard to the propulsion system we should bear in mind shaft lateral vibrations in two planes simultaneously. The difference between two plane lateral vibration and whirling vibration consists in shaft section proper rotation only.

Orbits of shaft surface point depending on shaft and whirling angular speed and corresponding bending stresses diagrams are shown on Fig.2.7.

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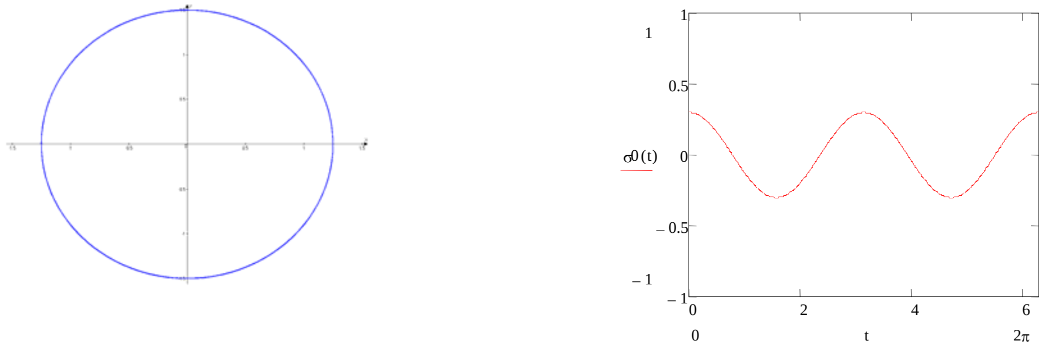
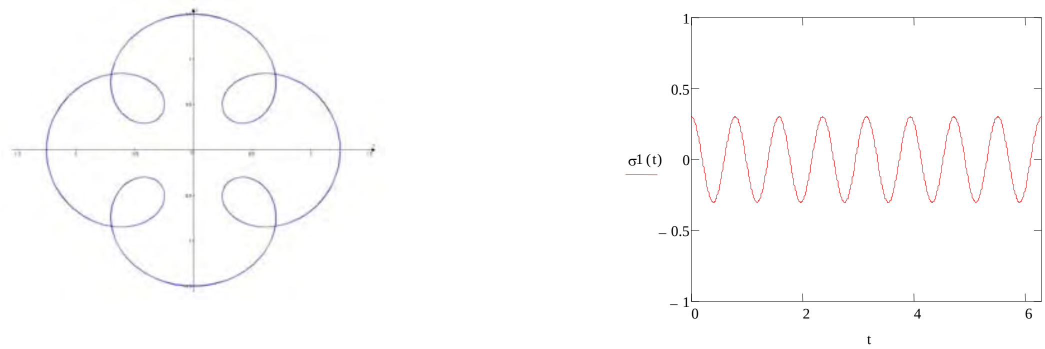
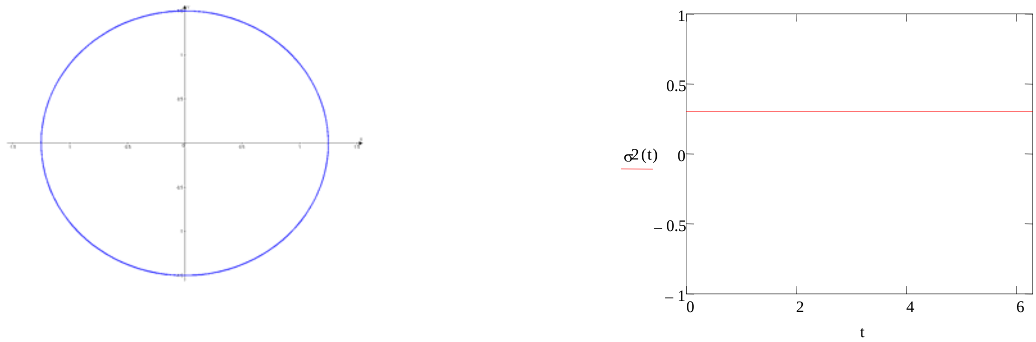
Lateral vibration in two planes ($\omega/\Omega = 0$ – non rotating shaft)



Forward asynchronous whirling ($\omega/\Omega = 5$)



Forward synchronous whirling ($\omega/\Omega = 1$)



Backward asynchronous whirling ($\omega/\Omega = -5$)

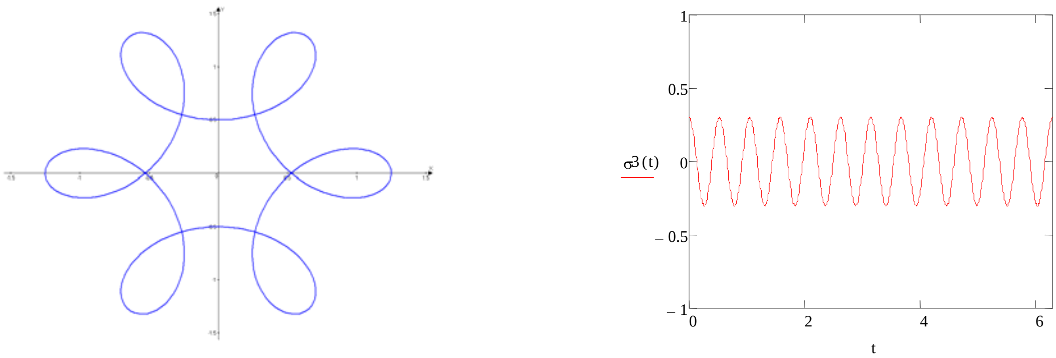


Fig.2.7 Lateral and whirling vibration motions of shaft surface point and bending stress

What is the source of whirling vibration? There are two classes of forces that excite whirling vibration in a propulsion system: internal and external. Where whirling vibration of propulsion system is excited by the internal forces there is no direct dependence between vibration intensity and transmitted power.
The alternating forces of both classes excite two kinds of whirling vibrations: forced vibration and self-excited vibration.
The main sources of propulsion systems whirling vibration are as the following:

- pulsating hydrodynamic forces onto propeller;
- radial excitation forces in reciprocating engines;
- pulsating lubrication pressures in plain bearings;
- friction forces in shaft material and couplings;
- unbalance;
- shaft misalignment due to assembly errors or manufacture defects;
- manufacture defects in gearing.

Excitation forces in more details are discussed hereafter.

Summarizing premises, the above mentioned linguistic ambiguity may be solved in the following way.

1. Vibration of non rotating propulsion shaft system should be referred as a lateral vibration.
2. If the shaft cannot loss contact with the point wise supports lateral vibration may be referred as a bending vibration.
3. Transverse vibration is a bending vibration in the horizontal plane.
4. Simultaneous lateral vibrations of non rotating propulsion shaft system in the vertical and horizontal planes may be considered as a whirling vibration.
5. Vibration of a rotating propulsion shaft system should be considered as a whirling vibration in any circumstances.

3. Gyroscopic effect in whirling vibration. Critical speeds

3.1 Gyroscopic effect

In a propulsion system, equipped with a heavy propeller, gyroscopic effect can influence the whirling vibration. When a huge mass with large polar and diametric inertia moments rotates gyroscopic moment M_{gyr} arises:

$$M_{gyr} = A \cdot I_d \cdot \Omega^2 \cdot \gamma ,$$

where:

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$A = 1 - \frac{I_p}{I_d} S$;

I_p – polar inertia;

I_d – diametric inertia;

$S = \frac{\omega}{\Omega}$ – whirling factor (positive for forward whirling and negative for backward whirling);

ω – shaft rotation speed;

Ω – whirling speed;

γ – shaft slope at the propeller.

Gyroscopic moment can be positive and negative depending on inertia moments relation and whirling factor value (Fig.3.1). Negative gyroscopic moment counteracts shaft deflection i.e. makes shaft 'stiffer'. Positive gyroscopic moment contributes to shaft deflection i.e. makes shaft 'softer'. So shaft natural frequencies of the rotating shaft with a propeller differ from natural frequencies of the non rotating shaft.
As far as it is not known in advance what type of whirling will occur in a propulsion system, natural frequencies for both forward and backward whirling shall be calculated. Natural frequencies for both types of whirling, calculated depending on shaft rotation speed, usually are shown on the diagram (Fig.3.2) introduced by Wilfred Campbell in 1924.

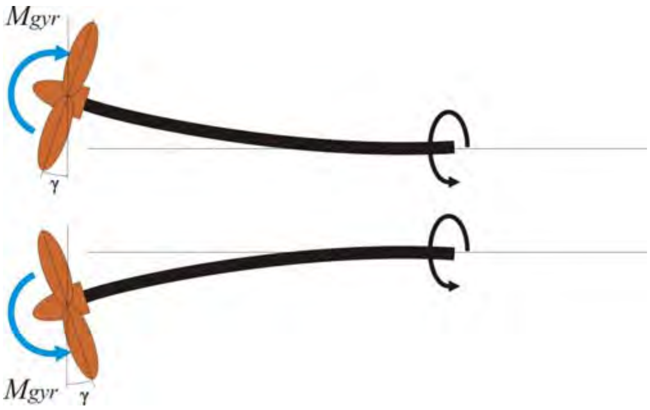

Fig.3.1 Positive and negative gyroscopic moments

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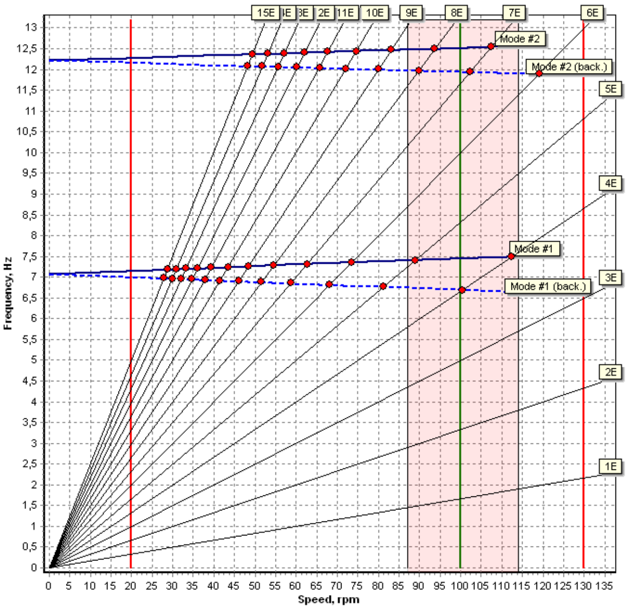

Fig.3.2 Campbell diagram of the rotating propulsion shaft

Straight lines on the Campbell diagram correspond to harmonic excitation orders. First order excitation is a once-per-revolution excitation. Four blade propeller excites the vibration of 4^{th}, 8^{th}, 12^{th} ... orders. The red points designate the possible resonances where the natural frequencies of whirling vibration coincide with the harmonic excitation frequencies.

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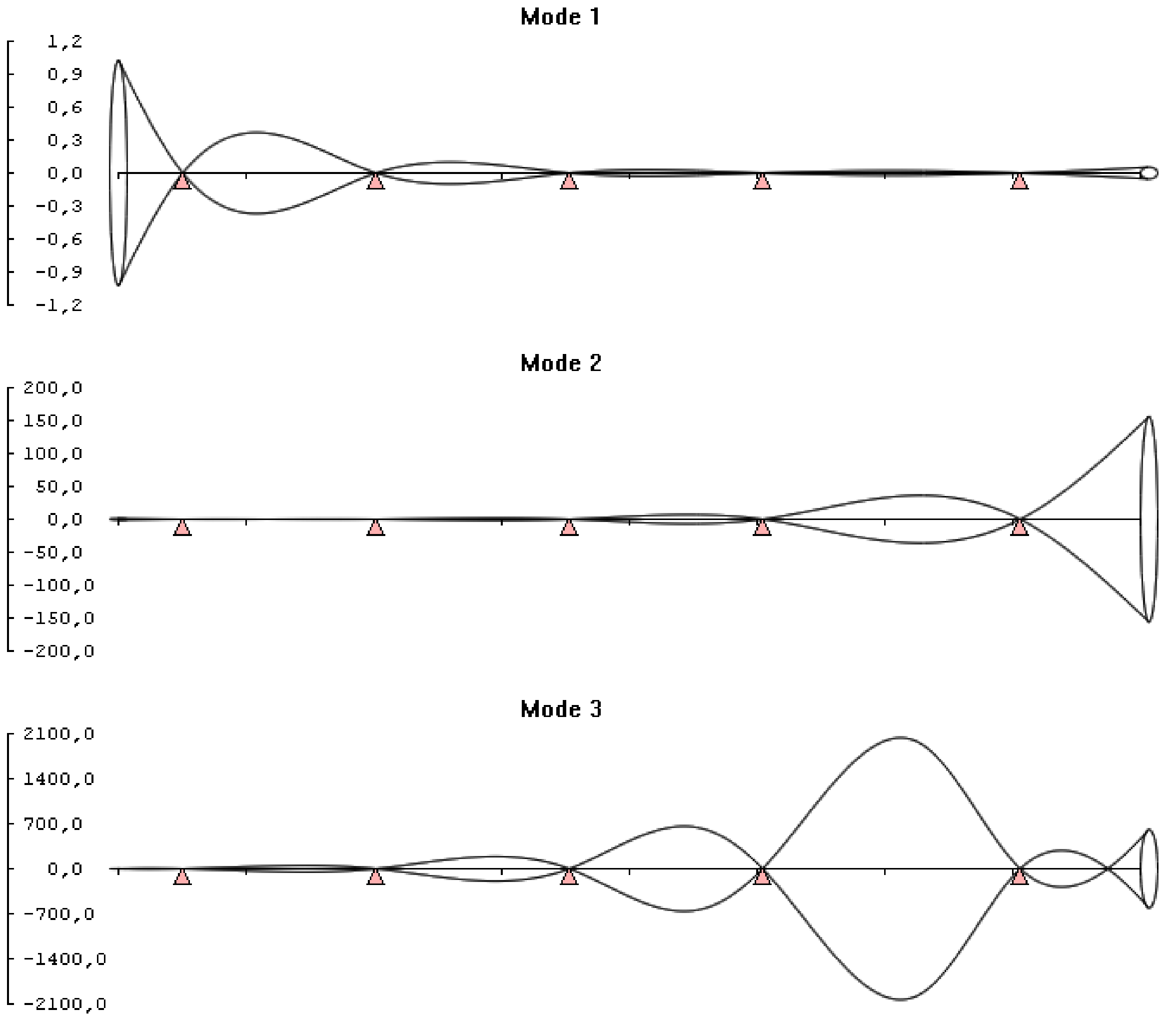

Fig.3.3 First three mode shapes of the multi-span beam whirling vibration

As far as vibration of non rotating propulsion shafts is of no interest for the shaft designers, natural frequencies and mode shapes should be calculated, taking into account the gyroscopic effect, especially for high speed propulsion systems.

3.2 Critical speeds

Speed of a rotating propulsion shaft at which severe vibration occurs is known as a ***critical speed***. Critical speeds coincide with the natural frequencies of the shaft whirling vibration.

Where shaft is supported by the bearings which have different stiffness in horizontal and vertical plane (anisotropic bearings) twice as many critical speeds exist (separate for vertical and horizontal vibration).

In spite of critical speed conception arose with regard to the first order unbalance vibration of the high speed rotors, now it is applied for any resonance vibration of rotating shafts irrespective of excitation force nature and vibration order.

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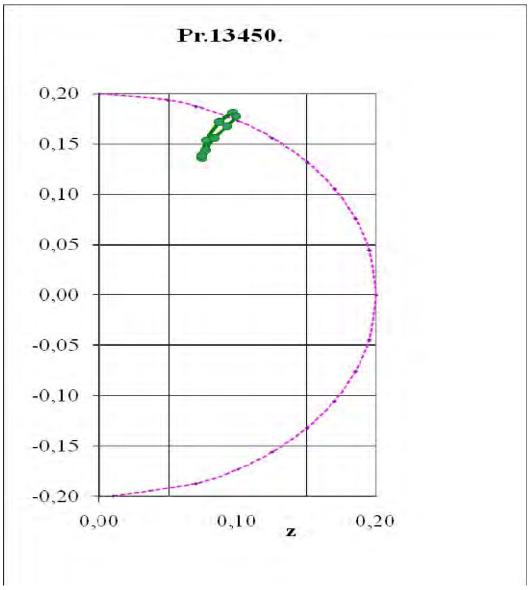
4.The sources of propulsion system whirling vibration

4.1 Propeller fluctuating loads

Hydrodynamics loads on propeller are main source of whirling vibration of a propulsion system and must be determined as exactly as possible.

Due to non uniform wake flow the effective hydrodynamic force is not directed along the shaft centerline. It travels along the closed path (Fig.4.1) and has some variable inclination angle to the propeller axis that results in variable vertical and horizontal forces F_{vert}, F_{hor} and moments M_{vert}, M_{hor},(Fig.4.2). Period of the thrust travel is equal to $(nZ)^{-1}$, where n is a rotation speed, Z is a blade number.



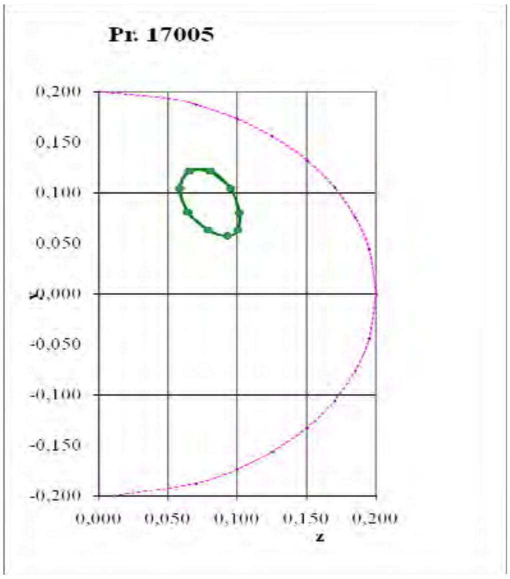


Fig.4.1 Locus of the effective thrust force

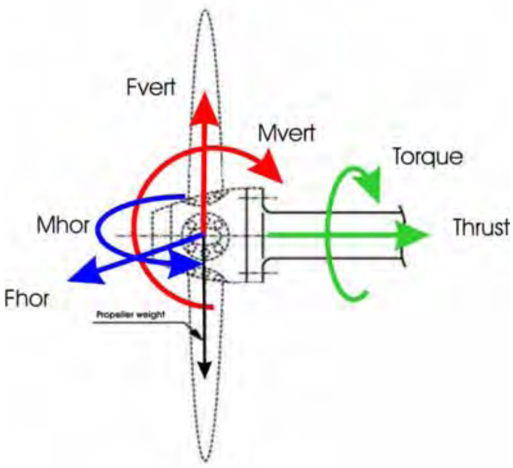


Fig.4.2 Propeller hydrodynamic forces and moments

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It should be noted that hydrodynamic forces and moments consist of a constant and fluctuating components:

$F = F_0 \pm F_f$,

$M = M_0 \pm M_f$.

Hydrodynamic propeller loads exerts bending stress in the propeller shaft. In the Fig.4.3 measured bending stresses in two propeller shaft sections are shown. Static bending stress amplitude due to propeller weight ($N = 0$) is shown as a horizontal line and does not depend of the rotation speed. The red line corresponds to the amplitude of the stresses exerted by the constant parts of hydrodynamic loads. The fluctuating components of the hydrodynamic loads excite intensive whirling vibration of the propulsion shafting at the range 75-95 rpm.

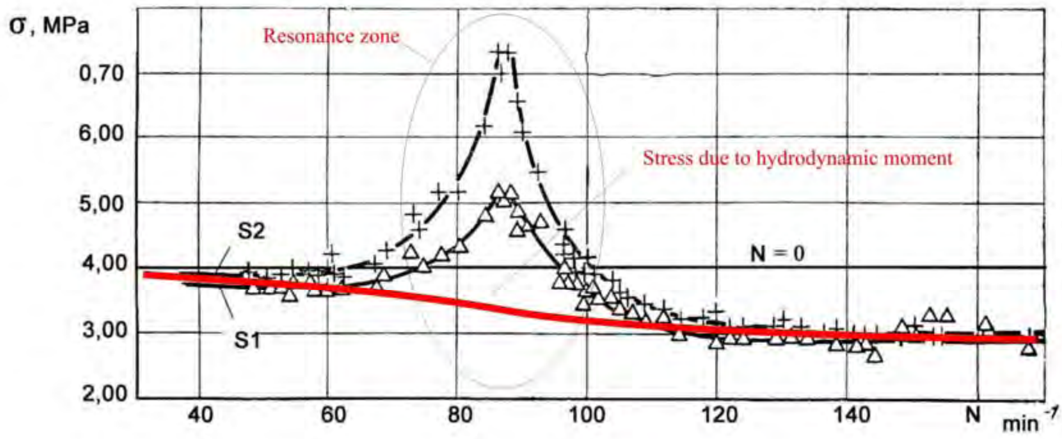
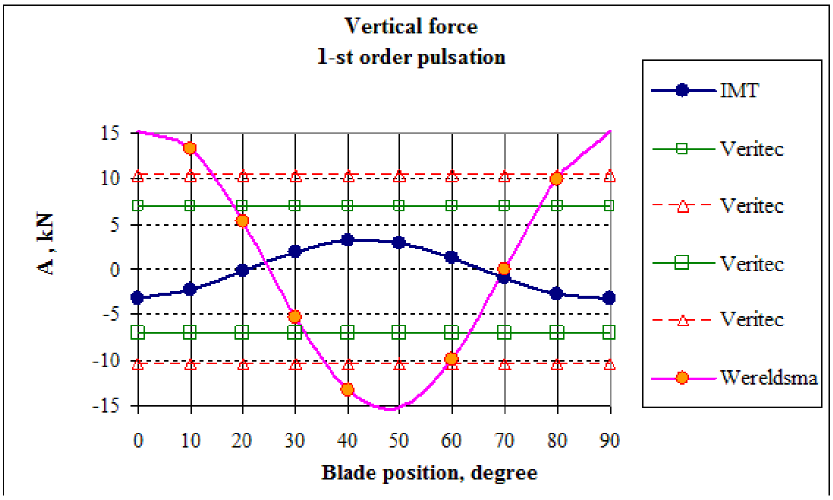
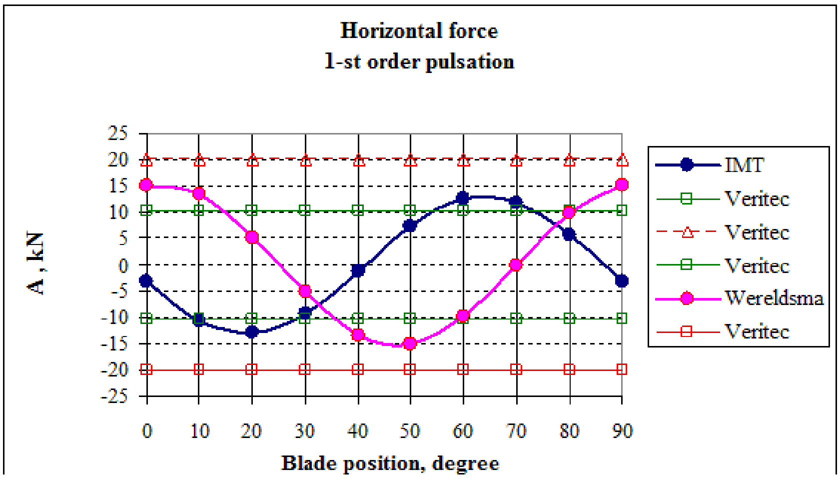
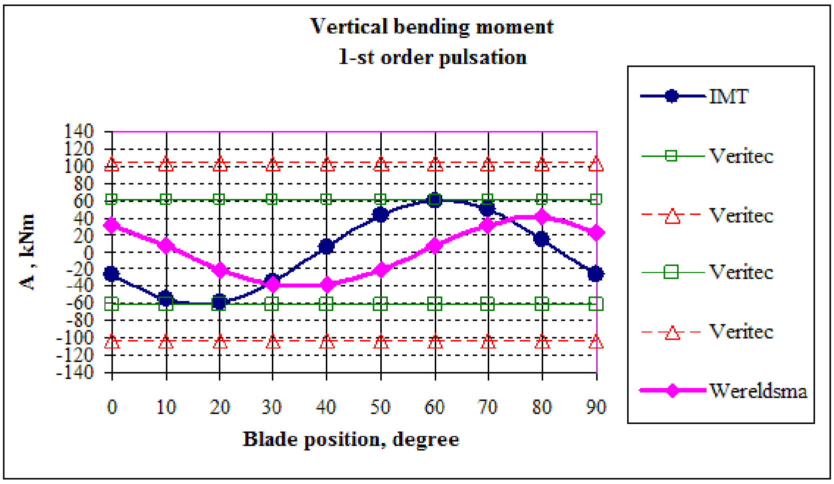


Fig.4.3 Whirling vibration stress measured in the propeller shaft [5]

State of art of propeller hydrodynamic load determination can be found in fundamental John Carlton book [6]. In the Fig.4.4 calculation results for first order pulsations obtained with the recommendations [7], [8] and with ShaftDesigner software are compared.







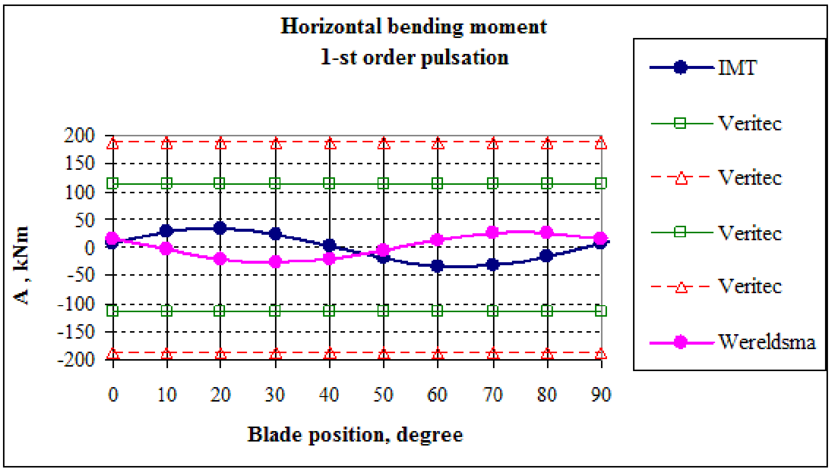


Fig.4.4 First order pulsating propeller loads

The fluctuating component of the thrust can excite propulsion shafting whirling vibration owing to an alternate bending moment produced by the thrust bearing. Thrust bearing absorbs the thrust in segments positioned over the lower 240 degrees of the

circumference of the thrust cam, see Fig. 4.5. Therefore the resultant reaction force is located somewhat lower than the shaft centerline.
Thrust eccentricity *e* equals 12÷14% of the average radius of the thrust bearing segments.

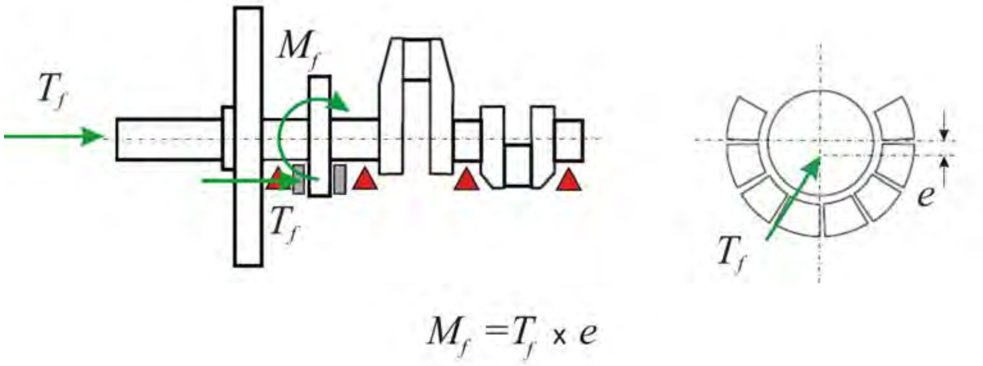


Fig.4.5 Fluctuating bending moment at the thrust bearing

The main problem of the hydrodynamic load calculations is the accuracy of input wake field parameters. It is very difficult to measure wake field parameters on a model as well as in full scale tests. So a very few experimental data have been published to use for theoretical methods calibration. In addition wake fields measured in the model tests without propeller are applied for hydrodynamic loads calculation of a full scale ship with propeller.

Such uncertainty in the excitation loads entail an inaccuracy of forced whirling vibration calculation.

4.2 Diesel engine excitation

Diesel engines excite whirling vibration in the same way as a torsional vibration is excited. For determination of torsional excitation tangential components are used while for whirling vibration determination radial components of gas pressures, inertia and weight should be used.
Most reliable data for radial components of gas pressures are supplied by engine manufacturers so the problems of whirling vibration excitation forces are the same as in the torsional vibration case.

4.3 Oil whirl and oil whip

Where high speed rotors is supported by plain bearings whirling vibration can be developed even there are no other external forces applied to the rotor. Below twice the first critical speed fluctuating lubrication pressures excite whirling with the speed equal to one-half of the rotation speed. It is called as an oil whirl or half speed whirl. Oil whirl is excited by circulatory forces in lubricating film.

If the rotor receives some sudden disturbing external impact, it can momentarily deviate from its equilibrium position. When this occurs, additional oil film pressure will drive the shaft into a whirling path around the bearing within the bearing clearance. Due to the damping forces, shaft can be returned to its stable position. Otherwise, shaft will continue the forward whirling motion with constant equal to critical speed. This self-excited whirling vibration is known as an oil whip. Oil whip start at twice of critical speed (mainly at low loaded bearings) and continue exists beyond that speed (Fig.4.6).

For high speed shafts the operation speeds are not to be near critical zones at $0.5\omega_{cr}$, ω_{cr} and $2\omega_{cr}$.

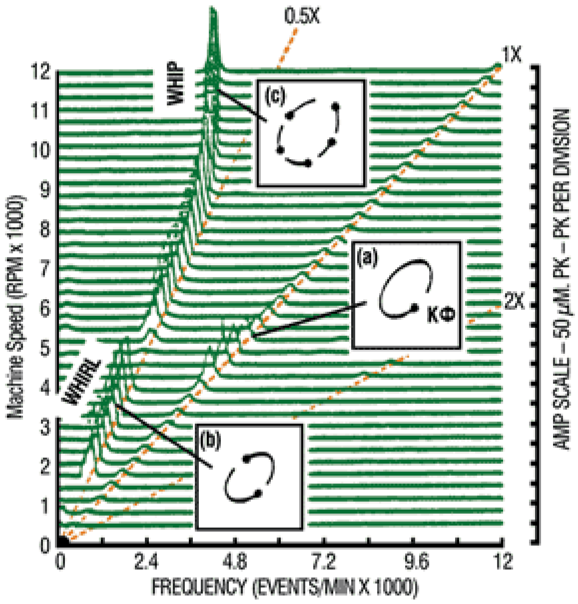


Fig.4.6 Oil whirl and oil whip in a high speed rotor
(www.machinerylubrication.com)

4.4 Hysteretic excitation

Whirling vibration due to hysteretic damping in shaft material or owing to friction in shaft couplings is self-excited vibration. The cause is internal friction forces when shaft rotate above the critical speed. Friction excites a backward whirling of high speed rotors.

4.5 Unbalance

Whirling vibration due to shaft static and dynamic unbalance is a forced vibration caused by internal centrifugal force.
It is well known that a propulsion shafting system will never be perfectly balanced. Due to material inhomogeneity the center of gravity does not coincide with the rotation axis. When the shaft rotates, the eccentric masses produce exciting centrifugal force. Unbalance excites forward synchronous whirling.

Unbalance excited whirling is most intrinsic for high speed rotors. First analysis of rotor whirling was performed by Rankine in 1869. Dunkerley published a paper (1895) describing an experiment results for turbine that operated at supercritical speed.

As to the convenient relatively low speed propulsion systems a whirling vibration due to unbalance can arise if shaft and propeller unbalance is considerable. Fortunately such unbalance is prevented by the special technological standards and survey procedures. Severe whirling vibration can arise after ship grounding or collision due to unbalanced propeller or bent shaft. But it is a casualty case.

4.6 Alignment-related errors

Alignment-related vibration of propulsion shafts is possible if the shafts are not properly coupled due to assembly or manufacture errors, Fig.4.7. It is the case when propulsion shafting loses the rotation axis continuity and smoothness. Alignment-related errors excite first order synchronous whirling vibration.

Fig.4.7 Propulsion shafting alignment errors

Excitations having alignment errors nature normally are not taken into account in whirling vibration calculation. Such errors must be prevented by adherence to the technological specifications and standards of shaft manufacture and alignment processes.

4.7 Manufacture defects in gearing

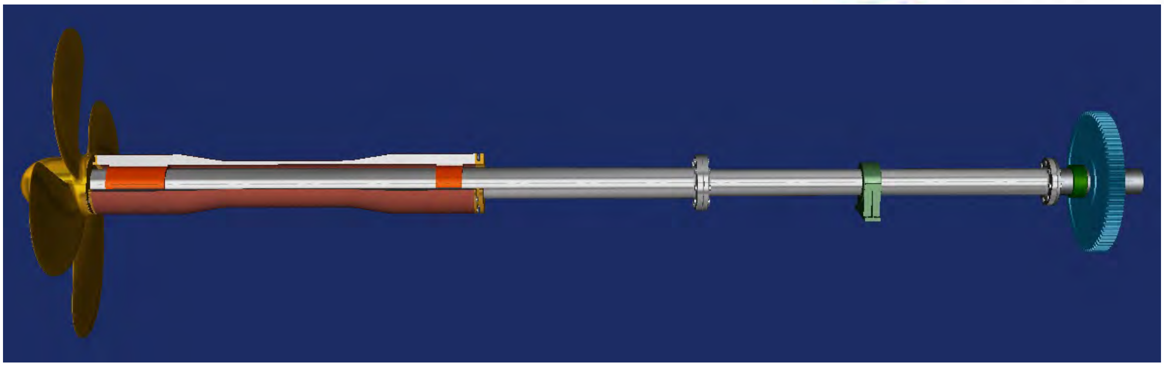
Manufacture defects in gearing are the possible source of harmful whirling vibration too. It should be taken into account when whirling vibration problems of geared installations are under consideration.

Gearing excited vibration can be easily identified due to high frequency of vibration. But similar to alignment-related errors it never been taken into account in whirling vibration calculation.

5.Propulsion shafting modeling

5.1 Shafts

For whirling vibration calculation Finite Element Method (FEM) is widely used. To use FEM real shafting system presents as a mass-elastic system where lumped mass are connected by weightless flexible elements Fig.5.1.



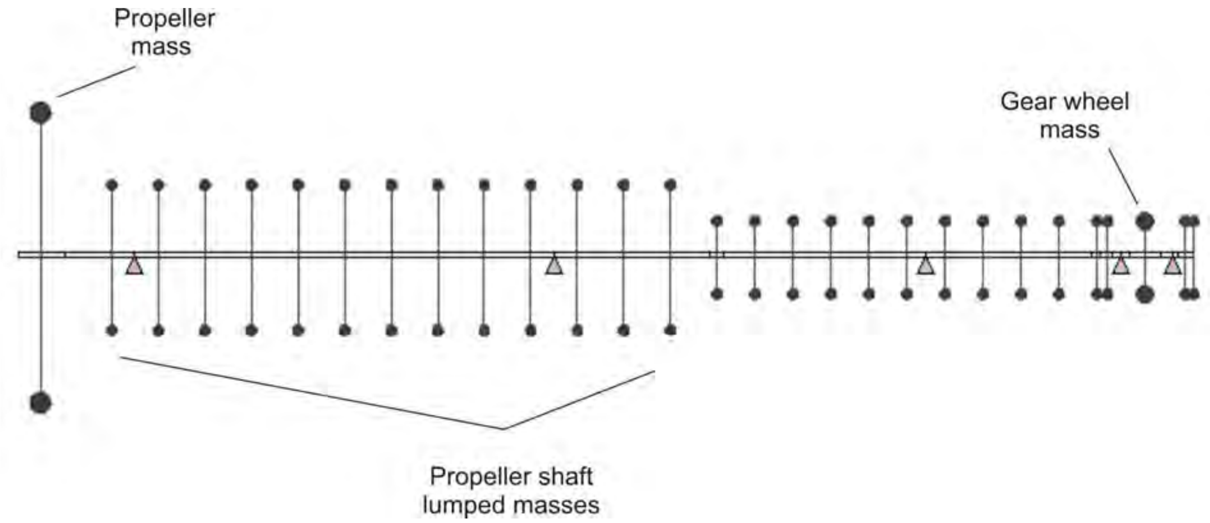


Fig.5.1 Propulsion shaft line an its FEM model

To build mass-elastic system shaft must be split onto elements – parts of the shaft where all geometry characteristics, material and environment are the same. Further elements refining may be required to provide appropriate calculation accuracy. If the system includes conical elements they are presented as a sequence of cylindrical elements.

Every shaft lumped mass has three characteristics, calculated as a half sum of the adjacent elements characteristics:

– mass M;
– diametric inertia moment I_d;
– polar inertia moment I_p.

Mass and inertia characteristics of the concentrated mass such as propeller, gear wheel must be added to correspondent lumped mass characteristics of appropriate nodes of mass-elastic system. Mass and inertia characteristics are the same for the vertical and horizontal bending.

Besides of dry propeller characteristics, hydrodynamic added mass and hydrodynamic added inertia moments must be taken into account.

Every flexible shaft element has five characteristics:
- length L;
- cross section inertia moment I ;
- effective section area $A_e = 0.9A$ (A full section area);
- module of elasticity E;
- Poisson's ratio μ.

Section and material characteristics are the same for vertical and horizontal bending. When the propeller shaft is lined for sea water corrosion protection (Fig.5.2) element characteristics are calculated using liner dimension and material properties.

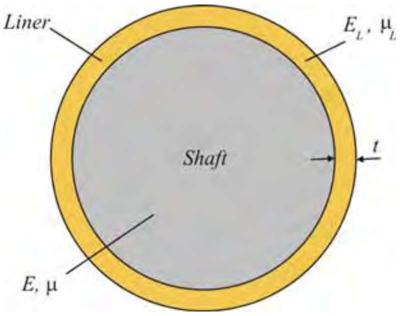


Fig.5.2 Cross section of lined element

Inertia moment of a lined element:

$$I = \frac{\pi}{64}\left[d^4\left(1-\frac{E_L}{E}\right)+\left(d+t\right)^4\frac{E_L}{E}\right]$$

When the shaft element is submerged in a liquid its mass must be increased by added mass. If the element of the propeller shaft is situated within a stern tube, added mass increases significantly dependent on shaft and stern tube internal radius relation r/R. Added mass Δm_0 of the circular cross section of radius r in free liquid of density ρ is

$\Delta m_0 = \rho\pi r^2$

In the Fig. 5.3 undimensional added mass coefficient μ, calculated by Alex Novosadosky. Increasing of propeller shaft mass will lower the critical speeds.

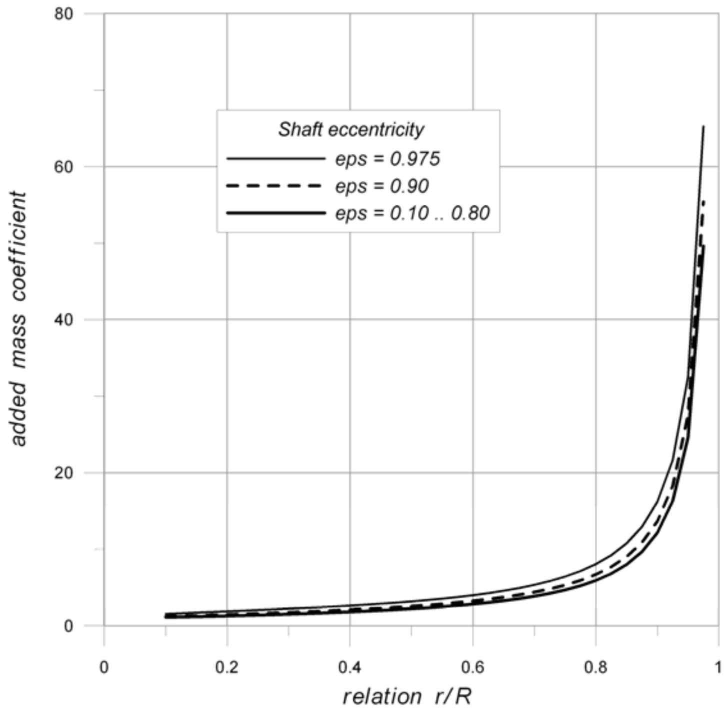


Fig.5.3 Cross section of lined element

Geometric characteristics of short shaft elements with abrupt changing of diameters (such a shaft flange) must be reduced, because element's material is not fully involved in bending deformations, (Fig.5.4).

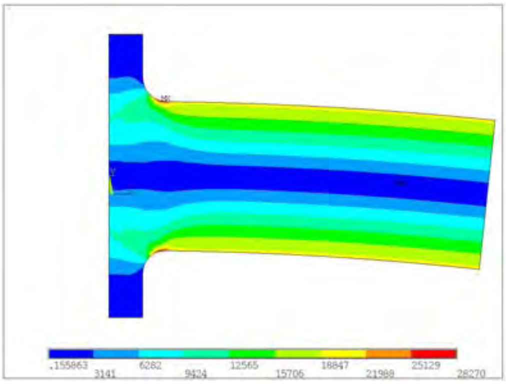


Fig.5.4 Bending deformations at the flanged coupling

Engine crankshaft is most complicate structure for mass and stiffness characteristics modelling. Engine manufacturers supply engine mass elastic system for torsional and axial vibration calculation but never for the lateral vibration.
In special software for diesel engine design 3-D solid FEM models of the crankshaft is used (Fig.5.5).

Fig.5.5 Crankshaft 3-D solid model

For the propulsion shafting vibration calculation so detailed modelling is excessive and leads to parametric vibration equation because elements inertia moments and element stiffness are dependent on shaft rotation angle. Fig.5.6 shows the dependence of main engine bearing loads on shaft rotation angle.

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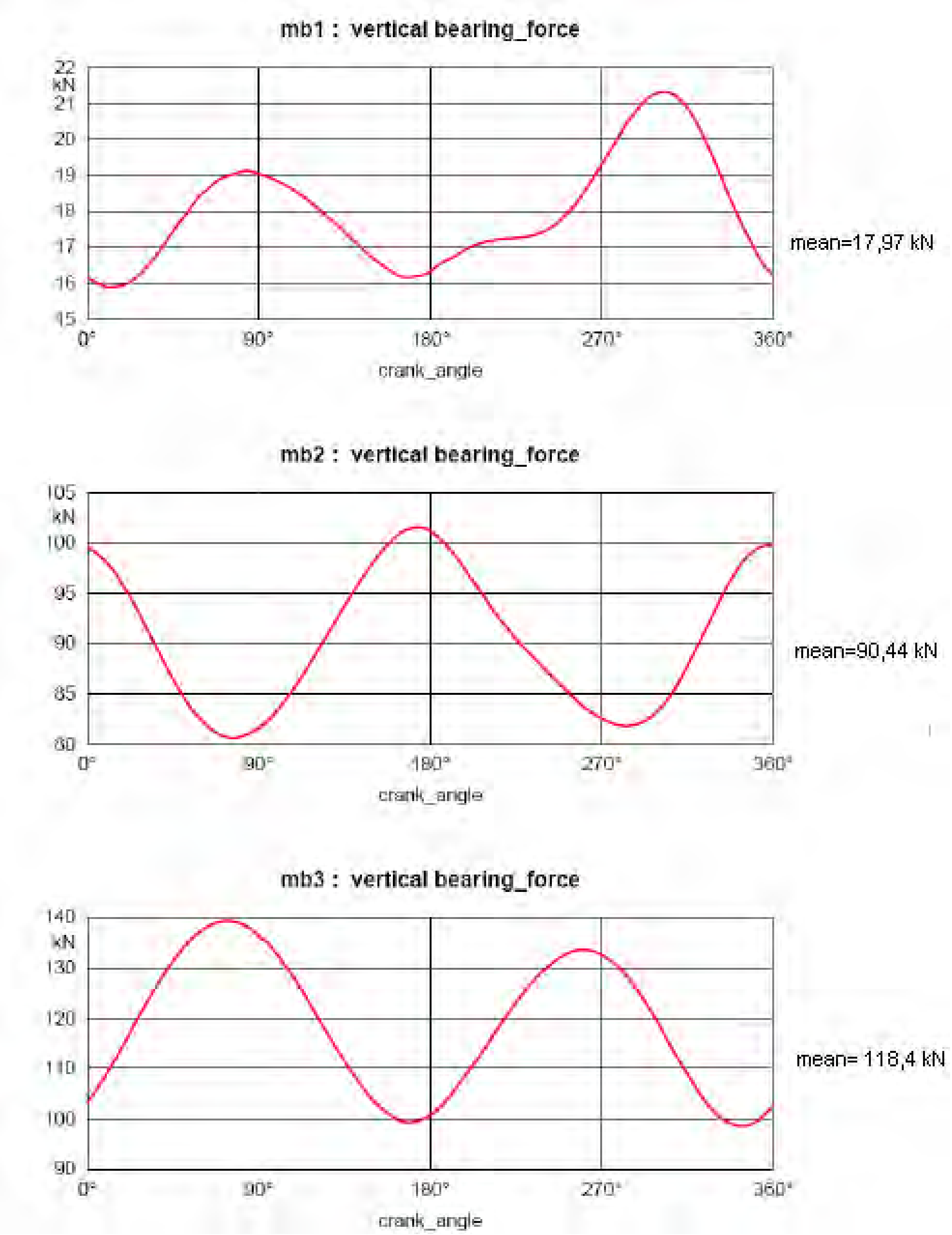


Fig.5.6 Varying static main bearing loads with one turn [9]

On the other hand due to the relatively short distances between engine bearings vibration amplitudes are small and modelling errors will not affect propulsion system whirling vibration significantly. So in whirling vibration calculation mean values of mass and stiffness characteristics of the crankshaft may be used.

5.2 Bearings modelling

In contrast to the torsional vibration whirling vibration deformations of the propulsion system are not monaxonic. Propulsion system undergoes to bending deformations in two planes and moves within the bearing clearance Fig.5.7. Shaft motion orbits measured ABS [10] within the aft stern tube bearing are shown on the Fig.5.8.

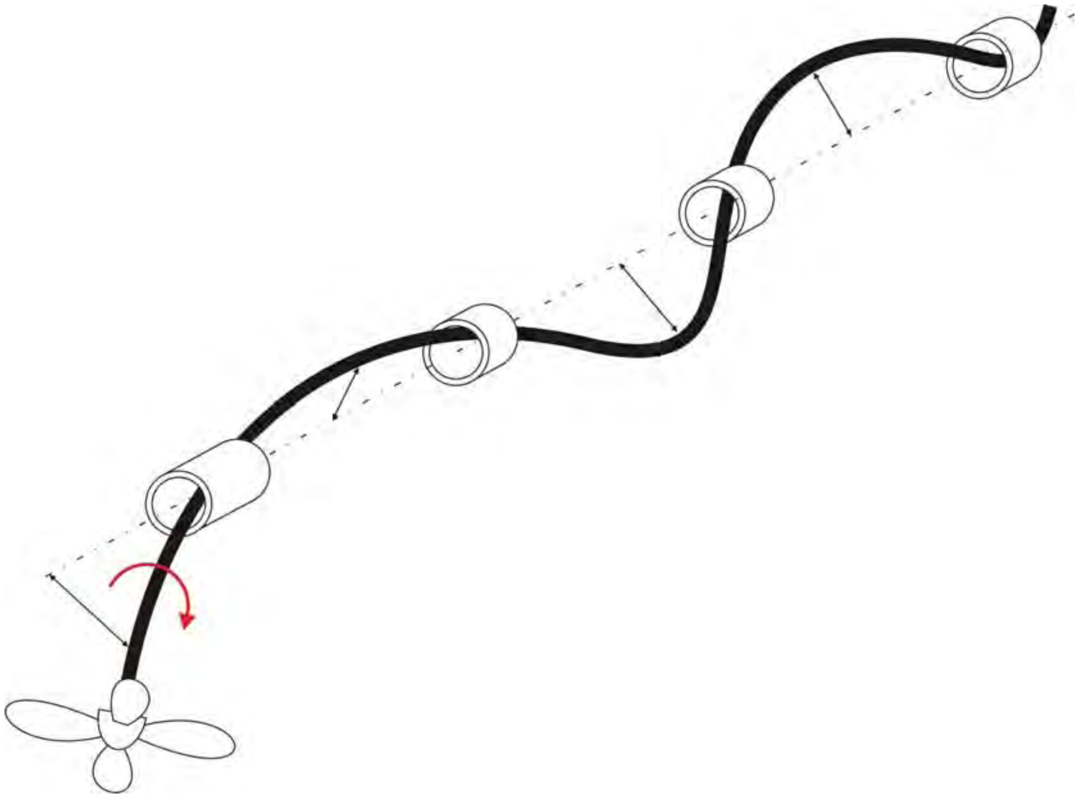


Fig.5.7 Shaft line whirling vibration motion

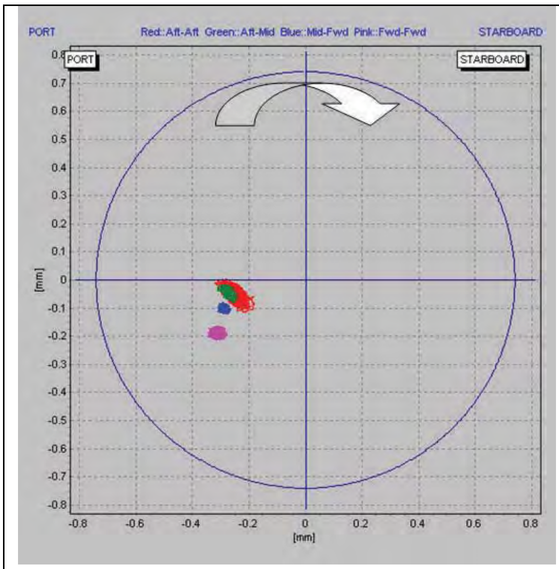
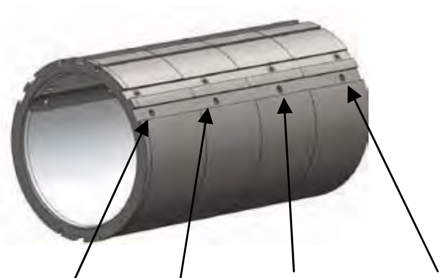
	Stable operation at 61 rpm  Red Green Blue Pink Red – location closest to the propeller

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Fig.5.8 Measured propeller shaft whirling orbits [10]

In torsional vibration calculation the propulsion system bearings are not of importance because all forces applied to the shaft line are self-balanced. When a whirling vibration is calculated boundary conditions of the shaft line must be taken into account. It means that not only shaft properties should be considered but the properties of shaft supporting structures, including their stiffness, geometry and space position.

Dynamic load $R(t)$ coming from the shaft involves all bearing structures in vibration motion. Bearing structure train (Fig.5.9) includes**:**

- lubricating film;
- bearing bush;
- bearing case;
- bearing stool;
- hull structure.

Propulsion shafting bearings are nor a point wise support nor an absolutely stiff support and influence whirling vibration significantly. It is safe to say that problem of bearing-shaft interaction modelling is the greatest bar to accurate calculation of propulsion system whirling vibration.

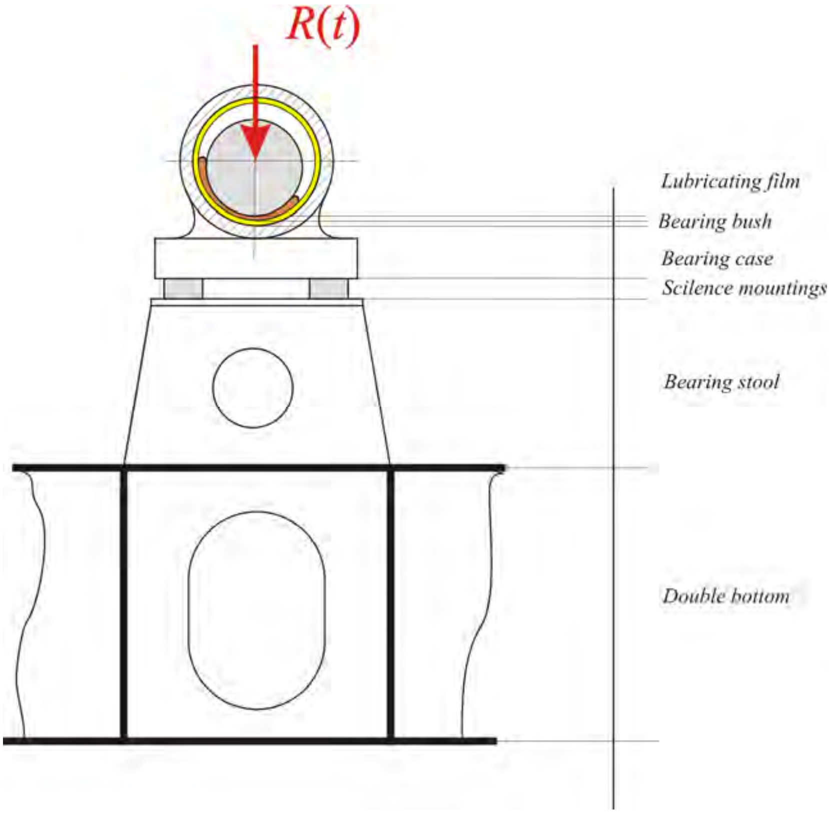

Fig.5.9 Bearing support structure

Owing to lubricating film properties the interaction of a bearing with shaft is strongly non-linear and direct using of a lengthy bearing model in practical whirling vibration calculation seems to be not realistic as well as detailed modelling of the whole bearing train. Currently it may be performed in the context of the research project only when propulsion system is modelled together with the bearing and hull structures using FEM.

During trial test of an aluminium mega-yacht a whirling vibration problem arose. To reveal the source of the problem, the company Techno Fysica BV developed a full FEM model. Half of the ship aft body is shown on Fig.5.10. The original shaft design whirling vibration calculation omitted the stiffness of the strut. When the real strut stiffness was entered in the whirling vibration calculation, the calculated frequencies became near the values measured during test trials.

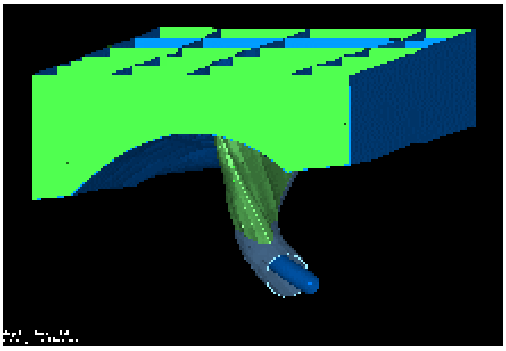

Fig.5.10 Strut support structure deformation

The simplest way to model bearing structure train in design calculation (harmonic excitation case) is to present it as a point wise anisotropic elastic support having a damping effect on shaft whirling vibration. It should be pointed out that the stiffness coefficients of the support as well as damping coefficients are frequency dependant. Moreover they are coupled due to the spatial motions of the shaft within bearing bush:

$$K_b(\omega)=\begin{pmatrix} k_{yy} & k_{yz} & k_{y\alpha} & k_{y\beta} \\ k_{zy} & k_{zz} & k_{z\alpha} & k_{z\beta} \\ k_{\alpha y} & k_{\alpha z} & k_{\alpha\alpha} & k_{\alpha\beta} \\ k_{\beta y} & k_{\beta z} & k_{\beta\alpha} & k_{\beta\beta} \end{pmatrix}, \qquad C_b(\omega)=\begin{pmatrix} c_{yy} & c_{yz} & c_{y\alpha} & c_{y\beta} \\ c_{zy} & c_{zz} & c_{z\alpha} & c_{z\beta} \\ c_{\alpha y} & c_{\alpha z} & c_{\alpha\alpha} & c_{\alpha\beta} \\ c_{\beta y} & c_{\beta z} & c_{\beta\alpha} & c_{\beta\beta} \end{pmatrix}.$$

Each matrix’s element consists of two components: lubricating film component and structural component. The main problem is a determination of these components.

It should be noted that currently for bearing stiffness and damping presentation simplified approach is usually used. Bearing stiffness and damping matrix are presented by two uncoupled coefficients:

$$K_b=\begin{pmatrix} k_{yy} & 0 \\ 0 & k_{zz} \end{pmatrix}, \qquad C_b=\begin{pmatrix} c_{yy} & 0 \\ 0 & c_{zz} \end{pmatrix}$$

Hereafter static and dynamic structural bearing stiffness k_{yy}, k_{zz} of chemical tanker (40000 DWT, 180 m length), calculated using Patran-Nastran software [11] are presented in the Tab.4.1.

static and dynamic structural bearing stiffness **Tab.4.1**

Excitation Point	Excitation direction	Static stiffness [MN/m]	Dynamic stiff-ness Excitation frequency: 7 Hz [MN/m]	Dynamic stiff-ness Excitation frequency: 10.5 Hz
Aft end of stern tube bearing	horizontal	850	2300	930
Intermediate bearing	horizontal	970	4700	1800
Fore engine main bearing	horizontal	4300	7300	3000
Aft end of stern tube bearing	vertical	910	470	1700
Intermediate bearing	vertical	1500	870	3600
Fore engine main bearing	vertical	3800	4100	4600

Lubricating film dynamic stiffness and damping k_{yy}, c_{yy} for stern tube bearing are shown in Tab.4.2. as a revolution function [11].

Lubricating film stiffness and damping **Tab.4.2**

Revolutions [rpm]	Dynamic stiffness [MN/m]	Damping [MNs/m]
30.0	69500	2900
50.0	31000	9500
76.0	25000	538
86.0	24600	471
95.0	24100	419

According to [9] lubricating film stiffness is much higher than those for the structural components.

It should be pointed out that lubrication conditions of propulsion shafting bearings have not yet been adequately investigated because until very recent time there was no software for calculation of propulsion shafting with plain bearings as a the single system.

5.3 Whirling vibration damping

5.3.1 Propeller damping

There are several methods for estimation of propeller damping in whirling vibration motion. Unfortunately any of them is recognized as a universal method. The simplest are Schwanecke's formulas [6] derived from unsteady propeller theory calculations:

$$c_F = 0.1536\frac{\rho\omega D^2}{\pi}\left(\frac{P}{D}\right)^2\left(\frac{A_e}{A_0}\right) \text{ kp s/m},$$

$$c_M = 0.1128\frac{\rho\omega D^4}{Z}\left(\frac{P}{D}\right)^2\left(\frac{A_e}{A_0}\right)^2 \text{ kp s},$$

where: c_F, c_M – propeller damping coefficients for linear and angular motions;

D – propeller diameter;

ρ – water density;

ω – propeller angular speed;

P/D – pitch ratio;

A_e/A_0 – blade are ratio.

Propeller damping is significant but its influence on whirling vibration parameters is not as decisive as in torsional vibration case. The propeller is installed in immediate proximity of the support (aft stern tube bearing) where motions amplitudes are not considerable.

5.3.2 General algorithm of bearing lubricating film stiffness and damping calculation

To find bearing lubrication dynamic stiffness and damping matrix in the case where no angular motions of the shaft are allowed:

$$K_b = \begin{pmatrix} k_{yy} & k_{yz} \\ k_{zy} & k_{zz} \end{pmatrix}, \qquad C_b = \begin{pmatrix} c_{yy} & c_{yz} \\ c_{zy} & c_{zz} \end{pmatrix}$$

the following steps have to be done.

1. For the current positions of the bearing defined by shaft alignment calculation the Reynolds' equation

$$\frac{\partial}{R\,\partial\theta}\left(\rho h_0^{\,3}\frac{\partial p_0}{R\,\partial\theta}\right)+\frac{\partial}{\partial x}\left(\rho h_0^{\,3}\frac{\partial p_0}{\partial x}\right)=6\mu\omega R\frac{\partial(\rho h_0)}{R\,\partial\theta},$$

has be solved in iterations to find the solution: function $p_0(x,\theta)$, integrals of which satisfy to shaft system static requirements:

$$R_y = \int_F p(x,\theta)\sin\theta dF, \qquad R_z = \int_F p(x,\theta)\cos\theta dF.$$

In Reynolds' equation:

R – shaft radius;

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$h_0(x,\theta)$ – gap thickness between shaft and bearing bush;

ρ, μ – lubricant's density and dynamic viscosity;

ω – shaft angular speed;

x, θ – shaft surface point coordinates.

2. After the equilibrium solution is found additional dynamic lubrication pressure $p_*(x,\theta)$ has to be calculated from the following Reynolds' equation:

$$\frac{\partial}{R\,\partial\theta}\left(h^3\frac{\partial p_*}{R\,\partial\theta}\right)+\frac{\partial}{\partial x}\left(h^3\frac{\partial p_*}{\partial x}\right)=\left[6\mu\omega\cos\theta-3\frac{\partial}{R\,\partial\theta}\left(h^2\sin\theta\frac{\partial p_0}{R\,\partial\theta}\right)-3\frac{\partial}{\partial x}\left(h^2\sin\theta\frac{\partial p_0}{\partial x}\right)\right]u_y+$$
$$+\left[6\mu\omega\sin\theta+3\frac{\partial}{R\,\partial\theta}\left(h^2\cos\theta\frac{\partial p_0}{R\,\partial\theta}\right)+3\frac{\partial}{\partial x}\left(h^2\sin\theta\frac{\partial p_0}{\partial x}\right)\right]u_z+$$
$$+12\mu\sin\theta\,\dot{u}_y-12\mu\cos\theta\,\dot{u}_z\,,$$

where:

$p_0(x,\theta)$ – lubrication pressure for non vibrating shaft;

u_y, u_z – vibration motions;

$\dot{u}_y, \dot{u}_z$ – vibration velocities;

3. Finally dynamic stiffness and damping have to be calculated as the components of dynamic loads:

$$R_y^* = \int_F p_*(x,\theta)\sin\theta dF = -k_{yy}u_y - k_{yz}u_z - c_{yy}\dot{u}_y - c_{yz}\dot{u}_z\,,$$
$$R_z^* = -\int_F p_*(x,\theta)\cos\theta dF = -k_{zy}u_y - k_{zz}u_z - c_{zy}\dot{u}_y - c_{zz}\dot{u}_z\,.$$

As can be seen from calculation algorithm, dynamic stiffness and damping coefficients are dependent on bearing shaft alignment parameters. In the case of harmonic excitation dynamic stiffness and damping are to be calculated for each frequency.

6. Whirling vibration parameters as a shaft alignment criteria

Two screw ship “Rotterdam” has two identical independent geared installations, Fig.6.1. Port side shaft line of the ship was run smoothly while the star board shaft line generated the violent vibration.

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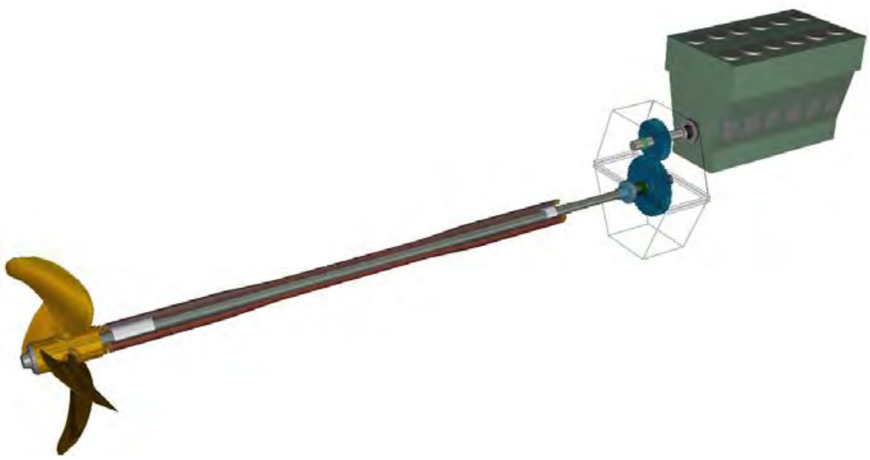

Fig.6.1 Propulsion shafting of the “Rotterdam” ship

Machine Support B.V. was invited by the owner to eliminate the problem. First of all shaft alignment parameters for both shaft lines were measured. The measurements revealed that the shaft lines were aligned differently, Fig.6.2, Fig.6.3.

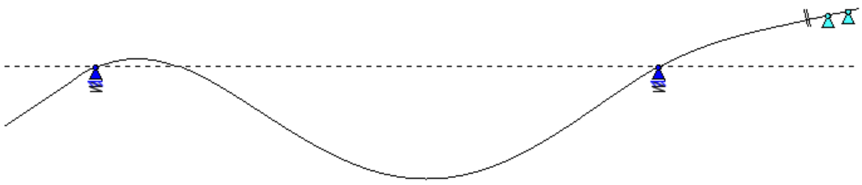

Fig.6.2 Alignment of the port side shaft line (non vibrating)

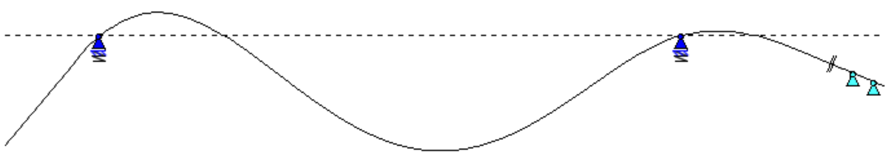

Fig.6.3 Alignment of the star board shaft line (vibrating)

After starboard shaft line was realigned in the same manner as the port side shaft line, harmful to ship structures and ship crew vibration disappeared.

This case distinctly shows the interconnection of shaft vibration and shaft alignment. Let us consider the possible causes of the interconnectivity.

Very often we can read or hear that the cause of an intensive vibration is a shaft alignment. It is very fuzzy statement. It is quite correct regarding to the short and rigid

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shafts such as motor and pump shaft, Fig.6.4. To have a perfect work they must be aligned geometrically before mating.

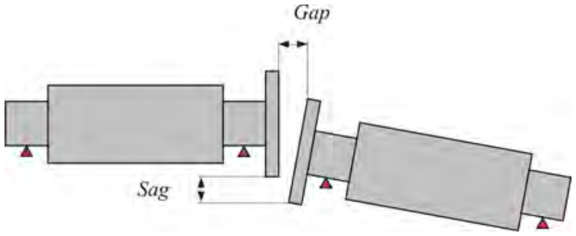

Fig.6.4 Misaligned motor and pump shafts

As to the propulsion system shafts they are always misaligned in the “motor-pump sense”, Fig.6.5.

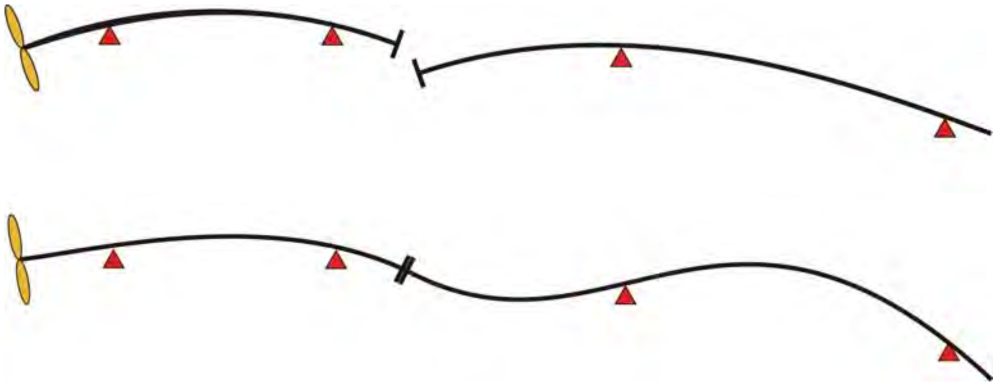

Fig.6.5 Misaligned propeller and intermediate shafts

But shaft flexibility compensates the misalignment and vibration does not arise.

The main cause of shaft alignment and shaft whirling vibration interconnectivity are the bearings.

Shaft alignment definition

To find an appropriate shaft alignment means to find the bearing’s position in space relative to chosen reference line. Bearing position in the space is defined by four parameters: vertical and horizontal offsets W_x , W_y of the bearing bush centerpoint and bush centerline tilt angles α , β in vertical and horizontal planes, Fig.6.6.

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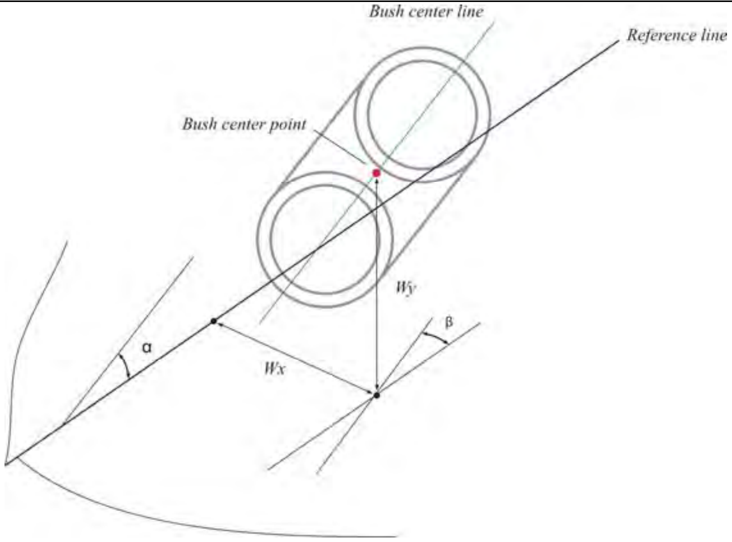

Fig. 6.6. Bearing alignment parameters

Bearings are not a point wise supports, they have a finite length. Therefore beam spans for vibration calculation purpose should be measured between the reaction points. These points migrate depending on bearing offsets, Fig.6.7.

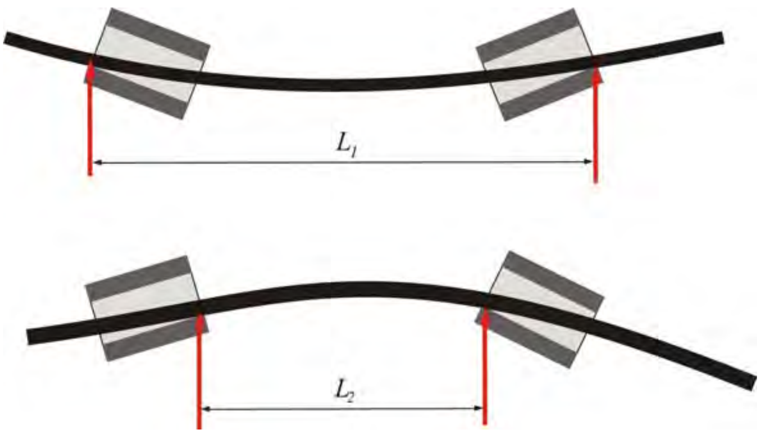

Fig.6.7 Beam span depends on shaft alignment

It is known that natural frequency of a single span uniform beam is inversely proportional to square of the beam span. So it will influence the natural vibration frequencies in the propulsion shafting case too.

As was stated before dynamic stiffness and damping characteristics of plain bearings considerably depend on lubrication film properties i.e. shaft alignment parameters (bearings positions). Samples of lubrication pressure distribution dependently on bearing position are shown Fig.6.8. It is quite reasonable that lubrication film properties will be different in each case.

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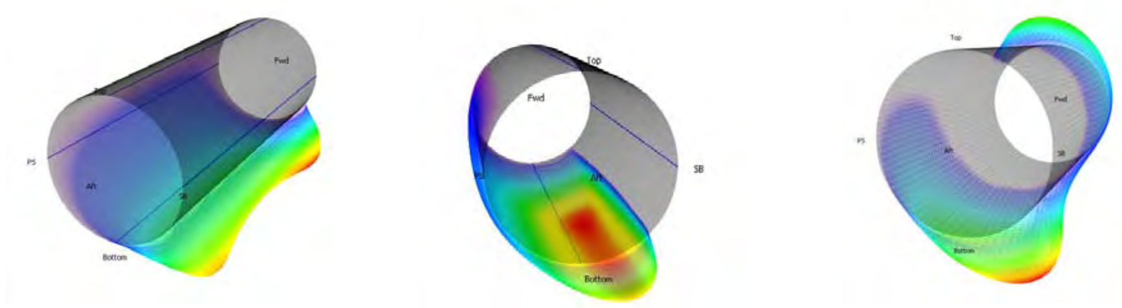

Fig.6.8 Samples of lubrication pressure distributions depending on bearing position

Inappropriate shaft alignment can place shaft in a position when the sum of the static load and additional dynamic load will exceed the permissible value. Bearing may be not reliably loaded when the difference of the static load and additional dynamic load is near to zero or negative. Such loading condition allows shaft to escape and start the whirl.

Therefore dynamic loads determined by whirling vibration calculation are to be applied in special alignment criteria to check the shaft alignment finally.

4 Conclusions

The uncertainties of the excitation loads and bearing dynamic stiffness and damping coefficients make impossible a rigorous prediction of whirling vibration parameters at present. For this reason some of Classification Societies reasonably require the critical speed calculation only. Forced whirling vibration calculations, if they are undertaken, in most cases are performed as parametric calculations.

Shaft designers should follow some general recommendations to prevent harmful whirling vibration:

– avoid having a long distance between the bearings. Often $L/d < 20$ is used as a criterion, where L is the bearing distance and d is the shaft diameter. It should be noted that, even when this criterion is satisfied, whirling may occur. This recommendation is in conflict with the shaft alignment calculation, where long distances are desirable;
– a bearing cannot be placed at the vibration node because such a bearing does not reduce the vibration displacements;
– avoid too flexible bearings and bearing supports;

– couplings should be near a bearing, e.g. a tooth coupling has no bending stiffness;
– large masses on free ends should be avoided. This means that the length of the propeller shaft aft of the aft bearing is critical.

REFERENCES

1. Henderson K. Analysing the causes of propulsion shaft failure. Marine Propulsion & auxiliary machinery. August/September 2010, pp 67-68.
2. Vartdal B.-J., Øvstaas J. Shaft line research enhances latest installation software. Marine Propulsion & auxiliary machinery August/September 2010, pp.58-59.
3. Bakken G.M. Stern bearing damage. DNV Machinery presentation. Marindagen Gøteborg, 2007.03.22.
4. Bantham I., Flicek P.F.G. Marine engineering failure investigation. Transactions of The institute of marine Engineers, v. 107, part 4, 1995, pp 257-270.
5. Luk'yanov I.S. Shaft alignment of propulsion shaftings of fishing vessels. Pischevaya promyshlennost', Moscow, 1979, p.368. (in Russian).
6. Carlton J. Marine Propellers and Propulsion – 2nd ed. Elsevier, 2007 533 p.
7. Wereldsma R. Propeller excited shaft and hull vibrations of single screw ships. International Shipbuilding Progress 11(1964), 124, pp 547-553.
8. Vibration control in ships. Veritec, Oslo 1985.
9. Bergande J. Alignment of direct-coupled large 2-stroke marine propulsion engines. Wartsila, 2006.
10. Šverko D., Šestan A. (2010) Experimental Determination of Stern Tube Journal Bearing Behaviour. Brodogradnja, 2, pp. 130-141.
11. Murawski L. Static and Dynamic Analyses of Marine Propulsion Systems Oficyna Wydavnicza Politechniki Warszawskiej, Warszawa, 2003, 138 p.